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## *In Prospect*

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Three questions were recently addressed by the Editor of the *Mathematics News Letter* to each of the forty-eight State Superintendents of Education in the United States and to the Superintendent of Education of the District of Columbia. Up to the present time twenty-nine replies of the forty-nine expected have been received. Long before the date of issue of *News Letter* No. 1, Volume 9, it is anticipated that all replies will be in the Editor's hands.

The three questions are:

1. Has mathematics as a required subject for graduation from high school been eliminated in your state?
2. If it has not been eliminated is there a definite prospect that it will be at an early date?
3. In your judgment are there good reasons for the hope that mathematics will have increased rather than diminished use in the secondary schools of your state?

Of the replies already received, many have generously exceeded the bounds indicated by the questions. Thus it appears probable that when all are in we shall have a basis for something like a measure of the nation-wide valuations of mathematics presently being made by all of the leading educational administrators in America. To what extent these valuations may be interpreted to mean a lowered or an unchanged rank of the premier science in the estimation of American public opinion it should be possible to determine with a considerable degree of accuracy by a careful gauge of these forty-nine expert official statements.

It is because this journal has always assumed, along with its other functions, the special function of serving the teacher of mathematics in every possible way that we make this editorial announcement of an early forthcoming analysis and interpretation of the data to be returned. The October issue of the *News Letter* should be anticipated with interest by those of its readers who are seriously concerned with the changed and changing status of mathematics in the secondary school. It is even possible in order that the analysis shall be thorough that it will be necessary to carry it into the November and December issues. **What an impressive gamut of opinion will be disclosed to the reader!**







has ever worked in this field with the same happy combination of deep mathematical problems and penetrating practical methods. In this connection should be mentioned his invention of the heliotrope and his calculation of Gaussian logarithms. During this period originated his work on cartographic projection—inspired by a prize question sent out by the Danish Academy.

Stimulated by Alexander von Humboldt from 1830 on, he participated in various magnetic experiments. In collaboration with the physicist, Wilhelm E. Weber, who was called to a Gottingen professorship in 1831, he succeeded in developing the theory of terrestrial magnetism into a new science. Gauss now showed how to present these results by means of a mathematical formula, the declination and inclination of the magnetic needle, the intensity at every point on the earth. He introduced the so-called absolute units (i. e., independent of time and place). The importance of determining these constants led to his discovery of the bifilar magnetometer and several other instruments.

His "Theory of Terrestrial Magnetism" appeared in 1839. Such men as Tobias Mayer and Hansteen had tried to explain this phenomenon on the basis of single magnets in the interior of the earth. Gauss conceived of a power which originates in a similar manner in the activity of magnetic molecules in the earth. Such forces are determined by their so-called potential, and he showed how this potential can be numerically calculated. Along with Laplace he became the founder of the theory of the potential. This work was of importance in later mathematical physics, as developed by Faraday, Maxwell, and Hertz, whose results led up to modern radio.

In this period falls his important research on the equilibrium of fluids, electrodynamics, and capillarity. Before the Royal Society of Gottingen he presented in 1843 his "Dioptric Studies on Systems of Lenses" and won a new harvest in a field which the unthinking believed had been already exhausted by Cotes, Euler, Lagrange, and Mobius. A most practical discovery had already occurred at Easter time, 1833. Gauss and Weber produced the first electromagnetic telegraph by stretching a wire 8,000 meters long, from the observatory to the physics laboratory. They had devised an alphabet, or code, and soon the later telegraphic advances in Europe and America were closely connected with this early development, especially the transatlantic line. As a physicist, Gauss may well be compared with Newton and Galileo.

Although so open and animated about these practical results, he was extremely reticent about his purely theoretical research, rightfully





question of current interest. We have progressed far enough now to make preliminary studies from available data, for example the quotations from records of the New York Stock Exchange. Such quotations indicate consolidated opinion as to what the progress of values should be before the history of a period actually records the values. One naturally wants to know what degree of accuracy may be expected of such predictions and to what extent one may be guided by purely mathematical considerations in dealing in stocks. The object of this paper is to illustrate a method of mathematical analysis applicable to such problems. A random sample of daily maximum fluctuations is taken from records of the New York Stock Exchange from March 1 to July 1, 1933 as reported by the Commercial and Financial Chronicle. For definiteness we will develop the problem under the following main divisions: The Sample; The Distribution, Mathematical Constants; A Theoretical Population; Conclusions.

*The Sample.* In selection of data it is necessary to so limit the population and so choose each datum that we may assume with a high degree of probability that the data represent a random sample of the population from which the sample is drawn. Observing that quotations are grouped as Railroads and Industrials and assuming that transportation is more indicative of general conditions than other values and are more nearly comparable than mixed quotations we select one thousand daily fluctuations in railway stocks as a sample. Equal numbers of items are taken each day ranging at random from the highest to the lowest priced stocks and from the most active to those for which there are no daily bids. In this way we seek to justify the assumption that the sample is typical of daily fluctuations for the period March 1 to July 1, 1933. Here the following questions are suggested.

1. What is a fair estimate of the expected value of a maximum daily fluctuation?
2. What is the expected variance of daily fluctuations?
3. What general form may be expected of the range of fluctuations?
4. What general principle should guide buying and selling in a rising market?
5. What is the probability that a fluctuation greater than a given amount may occur?

These are some of the questions considered in the following.



It is especially interesting at this point to consider the implications which attend the above values. In the first place if you plot the points indicated by class and corresponding frequency you will observe that a very large portion of the area under the polygon is below 10 with mode or highest point at two. With the range given in eighths of a point we see at a glance how small are a great majority of daily fluctuations. The arithmetic average computed from the value of  $\bar{u}$  is 9.892, or a little more than one point. The variance  $\frac{\mu_2}{2}$  being approximately one-third of the total range indicates a relatively high degree of concentration about the mean. The value .7 for skewness reflects the location of high frequency at the lower bound and the relatively long low spread of the sample to the right of the mean. The positive kurtosis is a significant value and is caused by the concentration of data into relatively few classes to left of the mean so as to produce a decided peak in the polygon. With these special characteristics the whole distribution is fairly smooth and regular. The chief value of these characteristics lies in the clue which they furnish to the problem of the next section in which we seek a general distribution of the population. We are now ready to attack the general problem of establishing a general formula or curve which represents the population from which the sample is drawn and to make estimates of the probable size of fluctuations.

*A Theoretical Population.* There is no mathematical criterion for selecting a formula or curve which best fits a given sample. The characteristic values which we have computed serve as a general guide. With a formula once selected it is general practice to determine the proper values of parameters by the so called method of least squares. By this we mean that the sum of squares of the differences between observed and corresponding theoretical frequency shall be a minimum. A more satisfactory result is sometimes attained by use of either the Pearson System or the Gram-Charlier System of frequency curves. Reference to the mathematical development of these systems may be found in the Notes of the Carus Mathematical Monograph, No. 3. Referring again to the values computed in the previous section we note the skewness with high frequency near zero as natural lower bound of the sample and the mode notably near zero. These characteristics suggest the Gram-Charlier Series, Type B, as a likely representative of daily market fluctuations. With this agreed we proceed to set up the



Taking this latter estimate we say that a daily fluctuation of as much as five points from the average would be expected to occur not more than 8 times in 1,000. A somewhat closer estimate may be found by moving the origin a distance  $h = \mu_2/d$  to the left of the mean and using moments about the new origin. However since we now know the probability is less than .008 we would hardly care to pursue the case further. As a check on the general problem one might try one of Pearson's Type curves and check the result with the above form of the Type B series.

*Conclusions.* For convenience we list certain observations which are suggested by the results of the paper.

1. One should remember that the sample used represents a rather rapidly rising market and the range of fluctuations is correspondingly wider than would ordinarily occur. Such renders the results more conservative for a normally fluctuating market.
2. The expected fluctuation of a stock taken at random from the market is slightly more than a point while the most probable value is one fourth of a point.
3. The Gram-Charlier Series, Type B, is the general form of the population of absolute values of maximum daily fluctuations in stock prices.
4. Generalized Tchebycheff formulas give fair approximations to the value of the probabilities for fluctuations of given value.
5. One is not justified in buying or selling stocks solely on a statistical basis. So many hidden influences affect sudden market fluctuations in individual stocks as to render estimates on the basis of the average inadequate for prediction of rise or fall. It does seem fair to conclude that one who has a particular stock to sell would be justified in acting on a fluctuation above the average. Or if one is ready to buy and has settled on the basic values of some stock a negative fluctuation more than the average would present a good probability of safety. Such conditions as consecutive runs in rise or fall are not included in the study.

## Book Review Department

Edited by  
P. K. SMITH

Felix Klein, *Elementary Mathematics from an Advanced Standpoint*. Translated from the third German edition (1924) by E. R. Hedrick and C. A. Noble. New York, Macmillan, 1932. 274 pages.

The writers of the textbooks in elementary mathematics, from arithmetic through calculus, have slurred over or omitted many essential steps in the logical development of their theory. It is possible for them to do this because our schooling methods encourage students to be receptive and uncritical. It is thought advisable for them to do this because the suppressed developments are too long and difficult, or too impractical, or too abstract to be appreciated by immature minds. It seems generally to be believed that, since a student's first grasp of a new idea is necessarily incomplete and inaccurate, an effective presentation of that idea must likewise be incomplete and inaccurate. The number system, especially irrational numbers, the postulational foundations of a theory, the simple properties of primes, exponents, logarithms, continuity and differentiability of functions, complex numbers, the existence of the numbers  $e$  and  $\pi$ , not to mention their transcendental character, impossible ruler and compass constructions, solutions of algebraic equations in terms of radicals, are but examples of topics which crop up in elementary mathematics and are treated like orphans.

Certainly there is practical justification for some measure of compromise of the strictly formal and logical method in teaching elementary mathematics. We must strike somewhere between the absurd extreme of beginning arithmetic with the postulational foundation of the number system and the other extreme of sinking into complete intellectual dishonesty by omitting all deductive proofs. Klein, in his *Elementary Mathematics From an Advanced Standpoint*, seems to think that we still lean too far toward the latter extreme. The book is written principally for young teachers of mathematics, but it is a book on the subject matter, not the pedagogy, of mathematics. The topics discussed include the list mentioned above. A topic like logarithms, for instance, is discussed as it is taught (as of 1908 or 1924 in Germany), as it developed historically, as it might be taught to elementary students, and as it is developed in modern function theory.

The book has a unique composition. Many of its statements are proved in completely rigorous detail. Other proofs are merely started

by the author who indicates the direction of further investigation and refers the reader to a reliable source. Formal proofs, however incontrovertible and elegant, do not satisfy the unsophisticated student when he asks *why*. Klein's rich historical introductions to various topics, his frequent appeal to intuition and to geometric perception help to provide the motivation which even the young teacher may need. And they make the book about as readable, in the popular sense, as accurate mathematics can be. It is thoroughly stimulating and could be read with pleasure and profit by any teacher of mathematics. The translators have done a notable service.

The principal drawback to the translation is that for completeness the book leans heavily upon other words, particularly Weber and Wellstein's *Encyklopadie der Elementarmathematik* and Tropfke's *Geschichte der Elementarmathematik* and these, alas, are still in German. Furthermore, the book was written years ago with the German school system in mind. Could not some American Mathematician of recognized ability do for American mathematics teaching a greater service by writing for the American teacher as Professor wrote for the German teacher?

W. L. DUREN, JR.  
Tulane University.

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### **Problem Department**

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*Edited by*  
T. A. BICKERSTAFF

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This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.



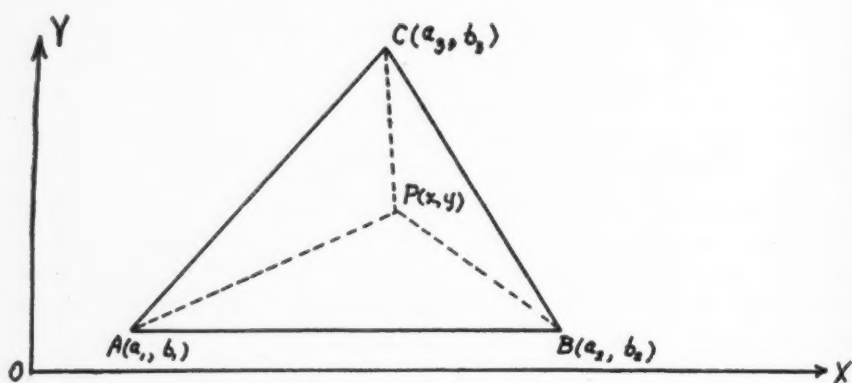
The point which solves Fermat's problem has the property that the sides of the given triangle subtend at this point equal angles, and on this account this point is often referred to as the "isogonic center" of the triangle (Roger A. Johnson, *Modern Geometry*, p. 221, Houghton Mifflin Co., 1929).

The isogonic center comes also to light in connection with the three equilateral triangles constructed on the sides of the given triangle as bases (Nathan Altshiller-Court, *College Geometry*, pp. 105-107; Johnson Publishing Company, Richmond, Va., 1925).

*A Solution* by H. F. S. Jonah, Purdue University, West Lafayette, Indiana.

Geometrically, we can show that the point P must lie in the plane determined by the three given points, if, we wish a minimum sum. We will further assume that the three given points do not lie on a line. In the case that the three points lie on a line one can easily determine the location of P.

We will, then, consider the problem: Being given a triangle ABC, to find a point P, in the plane of the triangle, such that the sum of the distances to the three vertices of the triangle is a minimum, i. e.  $PA + PB + PC = \min$ .



Then

$$1) \quad z = \sqrt{(x-a_1)^2 + (y-b_1)^2} + \sqrt{(x-a_2)^2 + (y-b_2)^2} + \sqrt{(x-a_3)^2 + (y-b_3)^2}$$











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